

3rd UP+C Integration Bee (Solucions)

Normes de correcció

- No cal constant, si hi ha una constant s'ignora a l'hora de puntuar
- Si el resultat és correcte (o ho sembla) però no està simplificat, demanar que es simplifiqui més, però no contar com a error
- Algunes simplificacions són subjectives, no cal ser massa estricte (sobretot amb logaritmes i potències)
- Anar molt en compte amb els signes, a vegades no es veuen bé al document
- $\log = \ln$
- Recordar coses com, per exemple, que nombres al denominador del logaritme poden sortir com a constant
- Sempre i quan el resultat no sigui numèric, les integrals hiperbòliques es poden expressar en termes de e^x

QUARTER-FINALS (4' per integral)

1.

$$\int_0^1 \sqrt{\frac{x}{1-x}} dx = \pi/2$$

2.

$$\begin{aligned} \int \sqrt{e^{4x} + 1} dx &= \frac{1}{2} \left(\sqrt{1 + e^{4x}} - \operatorname{arctanh} \left(\sqrt{1 + e^{4x}} \right) \right) \\ &= \frac{1}{2} \sqrt{1 + e^{4x}} + \frac{1}{4} \log \left(1 - \sqrt{1 + e^{4x}} \right) - \frac{1}{4} \log \left(1 + \sqrt{1 + e^{4x}} \right) \end{aligned}$$

3.

$$\int \sin(2x) \cos(3x) dx = \frac{1}{10} (5 \cos(x) - \cos(5x))$$

4.

$$\int \log(1 + \sqrt{x}) dx = -\frac{x}{2} + \sqrt{x} + (x-1) \log(\sqrt{x} + 1)$$

5.

$$\int (1 + 2x^2)e^{x^2} dx = e^{x^2} x$$

1.

$$\int \frac{1 + \cot(x)}{1 - \cot(x)} dx = \log(\cos(x) - \sin(x))$$

2.

$$\int \frac{1}{1 - e^{-2025x}} dx = \frac{\log(1 - e^{2025x})}{2025}$$

3.

$$\int 2025^{\log(x)} = \frac{x 2025^{\log(x)}}{1 + \log(2025)}$$

SEMI-FINALS 1 (4' per integral)

1.

$$\int_0^{\frac{\pi}{3}} \frac{|\sin(x) - \frac{1}{2}|}{\cos^2(x)} dx = 3 - \frac{3\sqrt{3}}{2}$$

2.

$$\int \frac{dx}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)^4} dx = -\frac{1}{2(\sqrt{\tan x} + 1)^4} + \frac{2}{5(\sqrt{\tan x} + 1)^5}$$

3.

$$\int \frac{dx}{x\sqrt{x^4 + 1}} dx = -\frac{1}{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{1 + x^4}}\right) = \frac{1}{4} \log\left(1 - \frac{1}{\sqrt{1 + x^4}}\right) - \frac{1}{4} \log\left(1 + \frac{1}{\sqrt{1 + x^4}}\right)$$

4.

$$\int_0^{\frac{\pi}{2}} \cos(x) \sqrt{1 - \sin 2x} dx = \frac{1}{2}$$

5.

$$\int_0^1 x \sqrt{\frac{1 - x^2}{1 + x^2}} dx = \frac{\pi}{4} - 1/2$$

1.

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\sin(x) + \sec(x)} = \frac{1}{\sqrt{3}} \log \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

2.

$$\int_{-\infty}^{\infty} e^{-x^2 - 4x - 5} dx = \frac{\sqrt{\pi}}{e}$$

3.

$$\int_0^1 x^3 \sqrt{1 + x^2} dx = \frac{2}{15} (1 + \sqrt{2})$$

SEMI-FINALS 2 (4' per integral)

1.

$$\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx = \frac{e^{x^2}}{2 + 2x^2}$$

2.

$$\int \frac{1}{(2x + 1)\sqrt{x^2 + x - \frac{3}{4}}} dx = \frac{1}{2} \arctan \left(\sqrt{x^2 + x - \frac{3}{4}} \right)$$

3.

$$\int \frac{\log(\tan(x))}{\sin(x)^2} dx = -\cot(x)(\log(\tan(x)) + 1)$$

4.

$$\int_0^\infty \lfloor x \rfloor e^{-x} dx = \frac{1}{e - 1}$$

5.

$$\int \frac{-2 - \log(x^2) + 3 \log^2(x)}{3 \log^3(x)} dx = \frac{x + 3x \log(x)}{3 \log^2(x)}$$

1.

$$\int_{-\infty}^\infty e^{-2x^2 - \frac{\pi x}{\sqrt{2}} - \frac{\pi^3 - 16}{16\pi} - \frac{1}{\pi}} dx = \sqrt{\frac{\pi}{2}}$$

2.

$$\int_0^2 \lfloor x^2 + 1 \rfloor \lceil x \rceil dx = 13 - 2\sqrt{2} - 2\sqrt{3}$$

3.

$$\int_{-2025}^{2025} \sqrt{|x|^{4048}} dx = 2 \cdot 2025^{2024}$$

FINALS (20' per fer 3 integrals)

1.

$$\int_0^1 x^{2024} \log^{2025}(x) dx = -\frac{2025!}{(2025)^{2026}}$$

2.

$$\int_{-\pi}^{\pi} \frac{x^2 \cos(x)}{1 + e^x} dx = -2\pi$$

3.

$$\int_1^{\infty} \left(\ln \left(1 - \frac{1}{x} \right) \right)^2 dx = \frac{\pi^2}{3}$$

4.

$$\int_{-1}^1 \frac{\tan^{-1}(x) \tan^{-1}\left(\frac{1}{x}\right)}{1 + x^2} dx = \frac{\pi^3}{48}$$

5.

$$\int_0^{\infty} \frac{dx}{1 + x^2 + x^3 + x^5} = \frac{\pi}{4}$$

6.

$$\int_0^{\pi} \frac{1 - \sin(x)}{\sin(x) + 1} dx = 4 - \pi$$

1.

$$\int_0^1 \frac{1}{(x+1)\sqrt{(x^2+x+1)}} dx = \tanh^{-1} 1/2$$

2.

$$\int_0^{\infty} \frac{dx}{\sqrt{1 + e^x + e^{2x}}} = -\log(-3 + 2\sqrt{3})$$

3.

$$\int \frac{e^{1/x}}{x^4} dx = \frac{-1}{x^2} e^{1/x} (1 - 2x + 2x^2)$$

4.

$$\int \frac{x^3 + x^2}{(2-x)(2+x)} dx = -\frac{1}{2}x(x+2) - 3\ln(2-x) - \ln(x+2)$$