Universitat Politècnica de Catalunya

3rd UPC INTEGRATION BEE

First round

Date: April 24th, 2025

Timing: 15:15 to 16:00 PM **Code:**

1.

$$\int_{-1}^{1} \left(1 - x + x^2 - x^3 + x^4 - x^5\right) dx = \frac{46}{15}$$

2.

$$\int \frac{2x+5}{x^2+1}\, dx = \log(x^2+1) + 5\arctan(x)$$

3.

$$\int_{0}^{\pi} xe^x \sin(x)\, dx = \frac{1}{2}(e^\pi(\pi - 1) - 1)$$

4.

$$\int (x+3)e^{x^2+6x+9}\, dx = \frac{1}{2}e^{(x+3)^2}$$

5.

$$\int e^x e^{e^x} e^{e^{e^x}} e^{e^{e^{e^x}}} e^{e^{e^{e^{e^x}}}} e^{e^{e^{e^{e^{e^x}}}}}\, dx = e^{e^{e^{e^{e^{e^x}}}}}$$

6.

$$\int_{2}^{e} \frac{\log^2 x - 1}{x \log^2 x}\, dx = -\frac{(\log(2) - 1)^2}{\log(2)}$$

7.

$$\int \frac{x^2+7x+8}{x^2+5x+3}\,dx = \log(x^2+5x+3)+x$$

8.

$$\int \tanh x - \tanh^3 x\,dx = \frac{1}{2}\tanh^2 x = -\frac{1}{2\cosh^2 x} = \frac{-2}{(e^{-x}+e^x)^2}$$

9.

$$\int_0^4 \lfloor x \rfloor^{\lceil x \rceil}\,dx = 90$$

10.

$$\int x^x(\sin(x)(1+\ln x)+\cos(x))\,dx = x^x\sin(x)$$

11.

$$\int_0^{\pi/2} \frac{\sin^4(x)+4\cos^4(x)}{\sin^2(x)+2\cos^2(x)-\sin(2x)}\,dx = 1+\frac{3\pi}{4}$$

12.

$$\int \frac{x^3+x^2}{\sqrt{x^3}+\sqrt{x}}\,dx = \frac{2x^{5/2}}{5}$$

13.

$$\int_0^1 \sqrt{x-\sqrt{x-\sqrt{x-\ldots}}}\,dx = \frac{1}{12}(5\sqrt{5}-7)$$

14.

$$\int \frac{e^x(x\sin(x)-\sin(x)+x\cos(x))}{x^2}\,dx = \frac{e^x\sin(x)}{x}$$

15.

$$\int_0^{2^{2025}\pi} \sin^{2025}(x)\,dx = 0$$

16.

$$\int \frac{2}{x^4\sqrt{x^2-25}}\,dx = \frac{2\sqrt{x^2-25}\,(25+2x^2)}{1875x^3}$$

17.

$$\int_0^{\infty} \frac{1}{\cosh x}\,dx = \frac{\pi}{2}$$

18.

$$\int \sin(x)e^{2\cos(x)}(\cos(x)-\sin(x)^2)\,dx = \frac{1}{2}\sin^2(x)\,e^{2\cos(x)}$$

19.

$$\int_0^{1} 2^{\lfloor \log_2 x \rfloor}\,dx = \frac{1}{3}$$

20.

$$\int \frac{1}{x\sqrt{x^{2025}-1}}\,dx = \frac{2\tan^{-1}\left(\sqrt{x^{2025}}-1\right)}{2025}$$

21. Bonus:

$$\int_0^{\infty} \frac{\arctan(x)}{x(\ln^2(x)+1)}\,dx = \frac{\pi^2}{4}$$

Instructions:
You have 45 minutes to solve 20+1 integrals.
No electronic devices or notes are allowed during the contest.
Only the final result will be taken into account for the final scores, and the intermediate steps in case of a tie.
It is not necessary to add the constant of integration to the answer, but it is mandatory to write the answer on the questions document.