

# 1st UP+C Integration Bee (Solucions)

## Normes de correcció

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- No cal constant, si hi ha una constant s'ignora a l'hora de puntuar
- Si el resultat és correcte (o ho sembla) però no està simplificat, demanar que es simplifiqui més, però no contar com a error
- Algunes simplificacions són subjectives, no cal ser massa estricte (sobretot amb logaritmes i potències)
- Anar molt en compte amb els signes, a vegades no es veuen bé al document (per exemple, el resultat de l'integral QF-5)
- $\log = \ln$
- Recordar coses com, per exemple, que nombres al denominador del logaritme poden sortir com a constant
- Sempre i quan el resultat no sigui numèric, les integrals hiperbòliques es poden expressar en termes de  $e^x$

## QUARTER-FINALS (4' per integral)

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1. 
$$\int_0^1 \frac{x}{1 + \sqrt{x}} dx = \frac{5}{3} - \ln 4 \quad (\text{tenir en compte diferents expressions del } \ln)$$

2. 
$$\int_{\pi}^{3\pi} \sin(\sin(x)) dx = 0$$

3. 
$$\int \sinh^2(x) \cosh^2(x) dx = \frac{\sinh(4x)}{32} - \frac{x}{8} = \frac{e^{4x}}{64} - \frac{e^{-4x}}{64} - \frac{x}{8}$$

4. 
$$\int \frac{x^3}{x-1} dx = \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(x-1)$$

5. 
$$\int \sin(2024x) \cos^{2022}(x) dx = -\frac{\cos(2023x) \cos^{2023} x}{2023}$$

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1. 
$$\int_{-\pi}^{\pi} \cos(2+x) \cos(2-x) dx = \pi \cos(4)$$

2. 
$$\int 2023^{2023x} dx = \frac{2023^{2023x-1}}{\ln 2023} = \frac{2023^{2023x}}{\ln(2023^{2023})} = \dots$$

3. 
$$\int_{-2023}^{2023} [x] dx = -2023$$

SEMI-FINALS 1 (4' per integral)

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1.

$$\int \frac{e^x + e^{-x}}{\sinh x} dx = 2 \ln(\sinh x) = 2 \ln(1 - e^{2x}) - 2x$$

2.

$$\int_{-\infty}^{\infty} e^{-\pi x^2 - 6\sqrt{\pi}x - 8} dx = e$$

3.

$$\int_0^1 \frac{2023}{x} \sqrt{\frac{x^{2023}}{1 - x^{2023}}} dx = \pi$$

4.

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx = \frac{22}{7} - \pi$$

5.

$$\int_0^\pi \frac{\max(\sin x, \cos x)}{\min(2^{\sin x}, 2^{\cos x})} dx = \frac{3 - 2^{1 - \frac{1}{\sqrt{2}}}}{\ln 2}$$

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1.

$$\int \frac{1}{x(x^2 + 1)} dx = \ln(x) - \frac{1}{2} \ln(x^2 + 1)$$

2.

$$\int_0^\infty \frac{\arctan(x)}{x^2 + 1} dx = \frac{\pi^2}{8}$$

3.

$$\int_0^2 \sqrt{1 - 2x + x^2} dx = 1$$

## SEMI-FINALS 2 (4' per integral)

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1.

$$\int \frac{1}{x \sin(\ln x)} dx = \ln \left| \tan \left( \frac{\ln x}{2} \right) \right|$$

2.

$$\int \frac{1}{x^3 + 1} dx = \frac{1}{3} \ln(x + 1) + \frac{\sqrt{3}}{3} \arctan \left( \frac{2x - 1}{\sqrt{3}} \right) - \frac{1}{6} \ln(x^2 - x + 1)$$

3.

$$\int \frac{1}{2 \cosh x} dx = \arctan(e^x) = \arctan \left( \tanh \left( \frac{x}{2} \right) \right)$$

4.

$$\int_0^\pi \frac{e^{\sec x}}{e^{\sec x} + e^{-\sec x}} dx = \frac{\pi}{2}$$

5.

$$\int_1^2 \max \left( \sqrt{1 - (x - 1)^2}, \sqrt{1 - (x - 2)^2} \right) dx = \frac{\sqrt{3}}{4} + \frac{\pi}{6}$$

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1.

$$\int_1^3 \frac{2^x}{2^{2^2-x} + 2^x} dx = 1$$

2.

$$\int \frac{\cos x - \sin x}{e^x} dx = \frac{\sin x}{e^x}$$

3.

$$\int e^x dx = \frac{e^{x+1}}{x+1}$$

FINALS (20' per fer 3 integrals)

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1.

$$\int_0^1 \ln^{2023}(x) dx = -2023!$$

2.

$$\int_1^{1+e} \ln(x - \ln(x - \ln(x - \dots))) dx = \frac{3}{2}$$

3.

$$\int_0^1 \frac{1}{7^{\lfloor \frac{1}{x} \rfloor}} dx = 1 + 6 \ln\left(\frac{6}{7}\right)$$

4.

$$\int_0^\infty \frac{\ln(x^2)}{x^2 + 2x + 2} dx = \frac{\pi}{4} \ln 2$$

5.

$$\int_0^1 \frac{dx}{(x+1)\sqrt{-x^2-2x}} = -\frac{i\pi}{3}$$

6.

$$\int_0^{\pi/2} \ln(\sin 2x) dx = \frac{-\pi \log 2}{2}$$

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1.

$$\int_0^3 [x]x^{[x]} - \lfloor x \rfloor dx = \frac{611}{12}$$

2.

$$\int \arcsin(2x) dx = x \arcsin(2x) + \frac{1}{2} \sqrt{1-4x^2}$$

3.

$$\int \frac{x+1}{(x+2)(x+3)} dx = 2 \ln(x+3) - \ln(x+2)$$